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# Magic maths moments: The power of dynamic and effective numeracy classrooms

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## Abstract

Encouraging students to think mathematically using open-ended questioning, investigation, reflection, and feedback is a key feature of effective numeracy classrooms. This paper describes how open-ended tasks are being used to build students' mathematical knowledge and skills alongside their abilities to inquire and think reflectively. In Tasmania, curriculum reform that values the development of higher-order capacities and the connection of knowledge and skills across the curriculum has enabled two teachers to work in a way that matches their own pedagogical beliefs and practices with respect to planning, teaching, and assessment for student learning.

## Introduction

Ron Ritchhart (2002) in his book, *Intellectual Character: What it is, Why it Matters, and How to Get it* challenges all teachers to consider their purpose for teaching. Is the role of teaching to fill students with knowledge, to help them succeed on national assessments, to prepare them for high school, or as Ritchhart would propose, are teachers “working to change who students are as thinkers and learners?” (p. 7). Developing classrooms as cultures of reasoning is critical in enabling students to build the intellectual capacities required of them in the twenty-first century.

Numeracy is an important aspect of the intellectual character of both students and adults. It has become an essential capability for any individual who wishes to participate fully in a democratic society, to apply not only knowledge and skills, but also critical reasoning capabilities, to learning and to everyday life. In Australia, a view of numeracy, beyond the ability to handle number concepts in context, is reflected by the Australian Association of Mathematics Teachers (AAMT) definition:

To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.

In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:

- Underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic);

- Mathematical thinking and strategies;
- General thinking skills; and
- Grounded appreciation of context. (AAMT, 1998)

Numeracy is about making meaning of mathematics, at whatever level of mathematical skill. It is about understanding and using mathematics, in all of its representations; for making sense of the world, for considering critically information presented, and for making informed decisions.

Researchers in both Australia and the United Kingdom have studied primary school mathematics with a view to identifying key practices that are indicative of highly effective teachers of numeracy. Askew, Rhodes, Johnson, and William (1997) described ‘connectionist’ teachers as those who emphasised the connections between ideas and concepts in mathematics and whose own beliefs and practices were based upon valuing students’ own methods. They encouraged students to describe their approaches and their thinking in solving problems. Clarke and Clarke (2002) similarly found that effective teachers of early years’ numeracy exhibited specific characteristics. These included the structuring of purposeful and open-ended tasks, tasks that focused on children’s mathematical thinking. In addition these teachers developed a culture of community in the classroom where students were encouraged to question, explain and share their thinking and evaluate others’ mathematical ideas. The teachers themselves reflected upon students’ responses, recorded observations, and used a variety of assessment methods.

Striving for excellence in the teaching of mathematics is multifaceted and involves ongoing learning for teachers with respect to their own knowledge, beliefs, and practices. In 2002, AAMT adopted a statement describing the knowledge, skills, and attributes considered to be important for excellence in the teaching of mathematics in Australian schools. Excellent teachers of mathematics do not only have a thorough and relevant knowledge of mathematics and of their students, and a commitment and enthusiasm for the profession, but they also make purposeful decisions that create for students a learning environment that maximises learning opportunities for each student. One aspect of the statement describes teaching in action as the capacity to “arouse curiosity, challenge students’ thinking, and engage them actively in learning” (AAMT, 2002).

## Learning Context

The use of open-ended tasks is one way to successfully create a learning environment that supports students in their learning of mathematics. Sullivan and Lilburn (2004) describe three main features of ‘good’ questions in the design of open-ended tasks:

1. They require more than remembering a fact or reproducing a skill.
2. Students can learn by answering the questions, and the teacher learns about each student from the attempt.
3. There may be several acceptable answers. (p. 2)

These types of questions encourage high levels of thinking and emphasise the value placed on students developing understanding of mathematical concepts so they can apply their knowledge and skills to new situations. Open-ended tasks also support learning that

assists students to follow arguments and be able to evaluate them and apply reasoned logic to any critique.

Two Tasmanian Grade 6 teachers are using open-ended tasks to support their students in the learning of mathematical concepts and skills, guided by reform curriculum in Tasmania that places thinking skills and strategies at the core of the curriculum and encourages the connection of knowledge and concepts across the curriculum. Tasmania's Essential Learnings Framework (Department of Education, Tasmania [DoE], 2002) is centred around five Essential Learnings: Thinking, Communicating, Personal Futures, Social Responsibility, and World Futures (Department of Education, Tasmania [DoE], 2002), and encouraging transdisciplinary activities. Other Australian states and territories are also reconceptualising curricula in terms of similar over-riding big ideas. Other reforms in Australia and internationally, although not necessarily systemic, are occurring within the mathematics classroom. These reforms, similarly, focus on the importance of conceptual understanding for students over procedural understanding, and encourage models of teaching and learning that improve students' capacities to reason, communicate, and reflect on their own learning experiences.

Three key elements of Tasmania's curriculum are driving the teaching of mathematics in these two Grade 6 classrooms: Being numerate, Inquiry, and Reflective Thinking.

*Being numerate.* The DoE embraces a comprehensive view of numeracy:

Being numerate involves having those concepts and skills of mathematics that are required to meet the demands of everyday life. It includes having the capacity to select and use them appropriately in real settings. Being truly numerate requires the knowledge and disposition to think and act mathematically and the confidence and intuition to apply particular mathematical principles to everyday problems... (DoE, 2002, p. 21)

*Inquiry and Reflective Thinking.* The central Thinking Essential Learning brings the processes of inquiry and reflection to all questions and investigations. In particular, the aim of the 'Inquiry' key element is to develop the capacity of learners to pose problems, gather relevant information, consider possibilities, make decisions, and justify conclusions.

Effective learners need the capacity to ask good questions, persevere in a line of inquiry, be systematic, set goals, and plan and follow a course of action... The ability to communicate what has been learnt and thought about, and to do so in a consistent, coherent, relevant and persuasive way, is essential in enabling learners to participate fully in schools, communities and workplaces. (DoE, 2002, p. 14)

## **Magic maths moments**

Three stories are included as told by the teachers. They provide examples of how open-ended tasks have been used by two teachers to support student learning in mathematics and resulted in moments that these teachers refer to as 'magic maths moments.'

## Square numbers as a bridge to understanding area and perimeter.

Every year I spend a lot of time working with students on square and rectangular numbers, including concepts of composites, primes, factors and multiples. I've never been convinced that students really understand these concepts as they seem to be forgotten quite quickly. I thought about a different way to approach it and tried rewording it as an inquiry task (see Figure 1):

**Do Numbers Have Shapes?**

Make a solid square (not just an outline) using counters.

How many counters did you use?

Can you make different sized squares?

What can you find out about the number of counters used for each square?

Can you make any other shapes?

Explore, record, and be prepared to share your findings.

*Figure 1. Do numbers have shapes?*

Within a very short time I could see 'light bulbs' switching on above kids' heads! Students quickly saw the connection between their squares of counters and certain number facts that they already knew. I was hearing comments such as:

"So that's why they are called square numbers!"

"I get it – I don't even have to make the squares because I've found the pattern."

"I don't have to count all the counters – it's 6 rows of 6 counters and I know that  $6 \times 6$  is 36. That means 36 is a square number!"

Students then went on to explore other shapes, not only rectangles and triangles but also circles and hexagons. Already they were 'stepping outside the square.' Many connections were made with previous knowledge, things they remembered but didn't really understand. The 'rectangular' group arrived at prime numbers by themselves when they were trying to find as many different rectangles as possible for each number between 2 and 20. At last the term 'prime number' was given a context and had some meaning:

"We can only make one rectangle using 2,5,7,11,13,17 and 19 counters. I think these numbers have a special name....?"

The 'triangular' group were really excited to discover a pattern, and could see why that pattern worked. They were able to accurately predict triangular numbers and spent a lot of time trying to find the biggest triangular number possible! (An emergency run to the maths resource room was needed when they wanted enough counters to check their prediction!). The 'circular group' made some amazing discoveries. It was a real learning experience for me as I had never considered circular numbers before and I'm not going to divulge their findings – you'll have to explore it yourself! The 'hexagonal group' were unable to construct regular hexagons with circular counters and after a while switched to one of the other shapes. This in itself led to some interesting discussion on the properties of shapes.

During the sharing back session some students were able to make the connection between what they'd just done and the concepts of area and perimeter. A fairly lively

discussion ensued with a number of students trying to transfer the  $L \times W$  formula used for finding the area of quadrilaterals to find the area of circles! The different mathematical concepts explored during this task are too numerous to mention but from then on whenever I mentioned square numbers I could almost see students visualising rows of coloured counters!

## Designing vegetable gardens and discovering $\pi$

As part of a unit investigating measurement the students were given the following problem:

### **The Vege Patch**

Tim wants to plant a vegetable garden but we have a lot of trouble with wallabies eating everything we plant.  
Tim bought a 60m length of very expensive wallaby-proof fencing wire to fence in an area of land where he could grow vegetables. He wanted to make his vegetable patch as big as possible.  
What is the largest possible area he can fence in with his 60m of wire?

*Figure 2. The vege patch*

Students tackled this problem in an amazing variety of ways. A large number of students drew diagrams showing the possible dimensions of the vegepatch. A lot of them started by setting out blocks in different arrays:  $20 \times 10$ ,  $15 \times 15$ , etc. Some students were able to quickly identify a pattern and completed the calculations without the use of blocks or diagrams:  $1 \times 29$ ,  $2 \times 28$ ,  $3 \times 27$ , etc. One group took a 60m length of string out to the school oval and measured out vege patches of different dimensions in order to see the relative sizes!

And then there was Tom...

Tom immediately saw the pattern and completed the calculations accurately. However his wasn't satisfied that a  $15 \times 15$  square was going to give Tim the greatest area because "we haven't considered any different shapes-what about triangles and circles?"

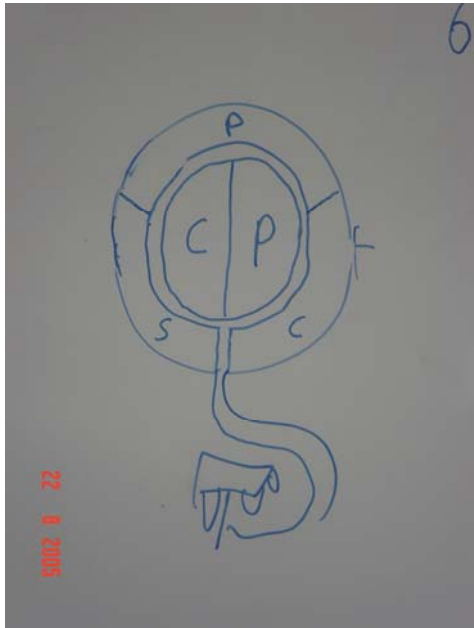
He decided to see if he could work out the area of a 60m circle. He measured the circumference of a circular table we had in the classroom. It was 4m. He covered the table in large sheets of paper and cut them to fit exactly. He then ruled a 5cm square grid on the paper. He counted all the squares and estimated how many whole 5cm squares could be made from the bits around the edge. He multiplied the number of squares by 5 to get the area of the table. He then multiplied this result by 15 to get the area of a 60m circle! Wow – this was all without any guidance from me and he was able to explain exactly what he was doing and why every step of the way!

BUT... Tom wasn't happy. He knew he did not have an accurate answer because of the bits of squares around the edge of the circle that he had had to estimate:

"Ms M, I've got as close as I can, but I suspect that you know an easier way to find the area of a circle!"

I wrote the term  $\pi$  on a piece of paper and suggested Tom might like to do some finding out about it. He 'googled' it and did a lot of reading. He discovered how to find the area of a circle given the diameter and/or radius and/or circumference. He then used a compass to construct circles of different sizes and test out his new found formulae! Once he was convinced that all was legitimate, he applied his newly acquired knowledge to his 60m circle and concluded that a circle was going to give Tim the biggest area for his Vege patch:  $\approx 286.6$  square meters!





Handwritten mathematical calculations on a chalkboard. The first line is  $60 \div 3.14 = D = 19.10828 \div 2 =$ . The second line is  $9.55414^2 \times 3.14$ . The third line is  $286.62419$ . The fourth line is  $225$ . The number '6' is written in the top right corner. A red date stamp '22 8 2005' is visible in the bottom right corner.

## Discussion

How powerful it is to unlock student thinking and to share – it changes the nature of the classroom and student learning.

Implications for assessment – when students have opportunity to use their own methods and explain their own thinking – teachers are provided with a depth of assessment opportunities. Misconceptions may be revealed, opportunities for further teaching therefore observational notes important assessment tool!

## References

- Clarke, D. & Clarke, B. (2002). Challenging and effective teaching in junior primary mathematics: What does it look like? In M. Goos & T. Spencer (Eds.), *Mathematics: Making waves*. (Proceedings of the 19<sup>th</sup> Biennial Conference of the Australian Association of Mathematics, pp. 309-318). Adelaide: AAMT Inc.
- Askew, M., Brown, M., Rhodes, V., William, D. and Johnson, D. (1997). *Effective teachers of numeracy*. London: King's College London.
- Australian Association of Mathematics Teachers. (2002). *Standards for excellence in teaching mathematics in Australian schools*. Adelaide: AAMT Inc.
- Department of Education Tasmania. (2002). *Essential learnings framework 1*. Hobart: Author. Mathematical Sciences Education Board & NRC
- Ritchhart, R. (2002). *Intellectual Character: What it is, why it matters, and how to get it*. San Francisco, CA: Jossey-Bass.
- Sullivan, P. & Lilburn, p. (2004). *Open-ended maths activities: Using 'good' questions to enhance learning in mathematics*. South Melbourne, VIC: Oxford University Press.